AFLEIDING

Cramer-Van Mises

$$W^2 = n\int_{-\infty}^{+\infty}\left(F_n(x)-F(x)\right)^2 dF(x)$$

x_i = i de orderstatistic. Verdeel reële rechte in

$(-\infty, x_1], [x_1, x_2], \ldots\ldots, [x_{n-1}, x_n], [x_n, \infty)$

$F_n(x)$ is een trapfunctie, constant tussen 2 opeenvolgende x_i-waarden en is gelijk aan $\frac{i}{n}$ tussen x_i en x_{i+1}.

$$\frac{W^2}{n} = \int_{-\infty}^{x_1}\left(F_n(x)-F(x)\right)^2 dF(x) + \sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}}\left(F_n(x)-F(x)\right)^2 dF(x) + \int_{x_n}^{\infty}\left(F_n(x)-F(x)\right)^2 dF(x)$$

$\Rightarrow$ Neem $u = F(x)$

$$\frac{W^2}{n} = \int_{0}^{F(x_1)}\left(\frac{0}{n}-u\right)^2 du + \sum_{i=1}^{n}\int_{F(x_i)}^{F(x_{i+1})}\left(\frac{i}{n}-u\right)^2 du + \int_{F(x_n)}^{1}(1-u)^2 du$$

$\Rightarrow$

$$\begin{aligned}
\frac{3W^2}{n} &= \left(\frac{1}{n}-F(x_1)\right)^3 - \left(\frac{0}{n}-F(x_1)\right)^3 \\
&+ \left(\frac{2}{n}-F(x_2)\right)^3 - \left(\frac{1}{n}-F(x_2)\right)^3 \\
&\cdots\cdots\cdots \\
&+ \left(\frac{n}{n}-F(x_n)\right)^3 - \left(\frac{n-1}{n}-F(x_n)\right)^3
\end{aligned}$$

Iedere term is van de vorm a^3-b^3 met $a-b=\frac{1}{n}$ $\Rightarrow$

$$a^3-b^3 = (a-b)(a^2+ab+b^2) = \frac{1}{n}\left\{3\left(a-\frac{1}{2n}\right)^2+\frac{1}{4n^2}\right\}$$

$\Rightarrow$

$$\left(\frac{i}{n}-F(x_i)\right)^3 - \left(\frac{i-1}{n}-F(x_i)\right)^3 = \frac{1}{n}\left[3\left(\frac{i}{n}-F(x_i)-\frac{1}{2n}\right)^2+\frac{1}{4n^2}\right]$$

$$= \frac{3}{n}\left(\frac{2i-1}{2n}-F(x_i)\right)^2+\frac{1}{4n^3}$$

$$\Rightarrow W^2 = \sum_{i=1}^{n}\left(F(x_i)-\frac{2i-1}{2n}\right)^2+\frac{1}{12n}.$$